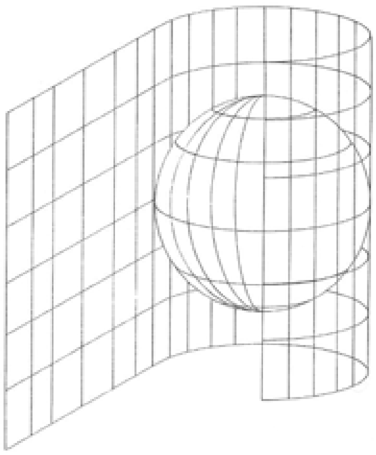
空間の

＜表面積＞と＜体積＞

計算資料



空間の残響計算をおこなう上で、＜表面積＞と＜体積＞の計算は必ずおこなわなくてはならないプロセスですが、その計算方法は以外と忘れがちです。

この資料は忘れ去った記憶を呼び戻すのにきっと役に立ってくれるものと思います。

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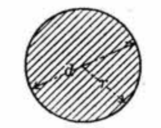
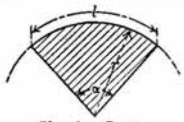
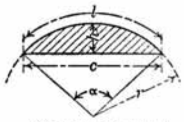
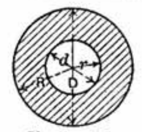
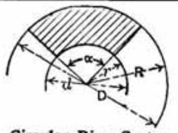
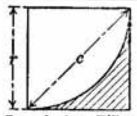
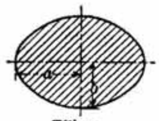
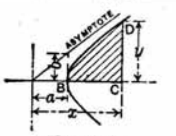
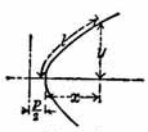
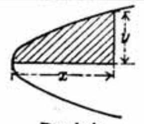
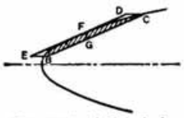
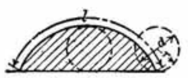
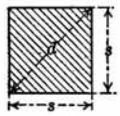
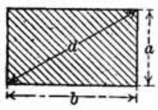
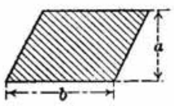
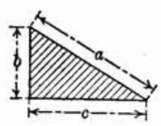
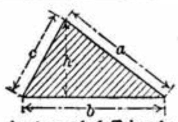
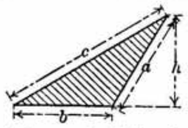
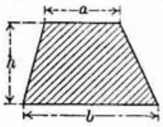
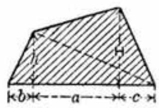
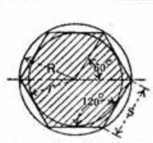
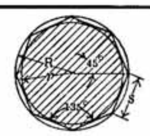
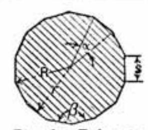
08/10/20

表面積の計算　その1

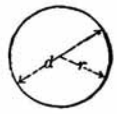
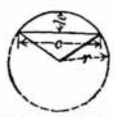
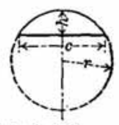
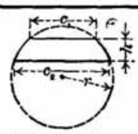
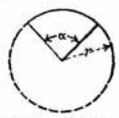
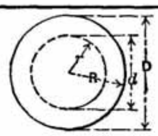
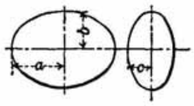
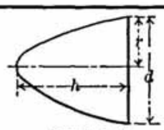
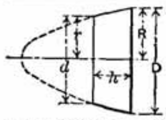
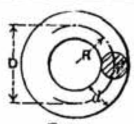
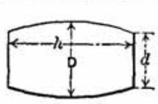
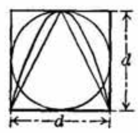
 Circle	A = area; C = circumference. $A = \pi r^2 = 3.1416\,r^2 = 0.7854\,d^2$ $C = 2\pi r = 6.2832\,r = 3.1416\,d$ $r = C \div 6.2832 = \sqrt{A \div 3.1416} = 0.564\sqrt{A}$ $d = C + 3.1416 = \sqrt{A + 0.7854} = 1.128\sqrt{A}$ Length of arc for center-angle of $1° = 0.008727\,d$ Length of arc for center-angle of $n° = 0.008727\,nd$
 Circular Sector	A = area; l = length of arc; α = angle, in degrees. $l = \frac{r \times \alpha \times 3.1416}{180} = 0.01745\,r\alpha = \frac{2A}{r}$ $A = \frac{1}{2} rl = 0.008727\,\alpha r^2$ $\alpha = \frac{57.296\,l}{r}$ $\quad r = \frac{2A}{l} = \frac{57.296\,l}{\alpha}$
 Circular Segment	A = area; l = length of arc; α = angle, in degrees. $c = 2\sqrt{h(2r-h)}$ $\quad A = \frac{1}{2}[rl - c(r-h)]$ $r = \frac{c^2 + 4h^2}{8h}$ $\quad l = 0.01745\,r\alpha$ $h = r - \frac{1}{2}\sqrt{4r^2 - c^2}$ $\quad \alpha = \frac{57.296\,l}{r}$
 Circular Ring	A = area. $A = \pi(R^2 - r^2) = 3.1416\,(R^2 - r^2)$ $= 3.1416\,(R+r)(R-r)$ $= 0.7854\,(D^2 - d^2) = 0.7854\,(D+d)(D-d)$
 Circular Ring Sector	A = area; α = angle, in degrees. $A = \frac{\alpha\pi}{360}(R^2 - r^2) = 0.00873\,\alpha\,(R^2 - r^2)$ $= \frac{\alpha\pi}{4 \times 360}(D^2 - d^2) = 0.00218\,\alpha\,(D^2 - d^2)$
 Spandrel or Fillet	A = area. $A = r^2 - \frac{\pi r^2}{4} = 0.215\,r^2$ $= 0.1075\,c^2$

 Ellipse	A = area; P = perimeter or circumference. $A = \pi ab = 3.1416\,ab.$ An approximate formula for the perimeter is: $P = 3.1416\sqrt{2(a^2+b^2)}$ A closer approximation is: $P = 3.1416\sqrt{2(a^2+b^2) - \frac{(a-b)^2}{2.2}}$
 Hyperbola	A = area BCD. $A = \frac{xy}{2} - \frac{ab}{2}$ hyp. log $\left(\frac{x}{a} + \frac{y}{b}\right)$
 Parabola	l = length of arc. $l = \frac{p}{2}\left[\sqrt{\frac{2x}{p}\left(1+\frac{2x}{p}\right)} + \text{hyp. log}\left(\sqrt{\frac{2x}{p}} + \sqrt{1+\frac{2x}{p}}\right)\right]$ When x is small in proportion to y, the following is a close approximation: $l = y\left[1 + \frac{2}{3}\left(\frac{x}{y}\right)^2 - \frac{2}{5}\left(\frac{x}{y}\right)^4\right]$, or $l = \sqrt{y^2 + \frac{4}{3}x^2}$
 Parabola	A = area. $A = \frac{2}{3}xy$ (The area is equal to two-thirds of the rectangle which has x for its base and y for its height.)
 Segment of Parabola	A = area. Area $BFC = A = \frac{2}{3}$ area of parallelogram $BCDE$. If FG is the height of the segment, measured at right angles to BC, then: Area of segment $BFC = \frac{2}{3}\,BC \times FG$
 Cycloid	A = area; l = length of cycloid. $A = 3\pi r^2 = 9.4248\,r^2 = 2.3562\,d^2$ $= 3 \times$ area of generating circle $l = 8r = 4d$

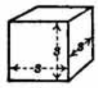
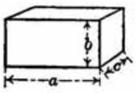
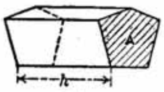
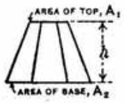
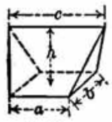
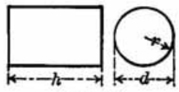
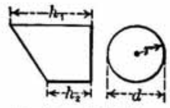
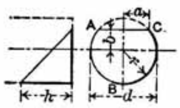
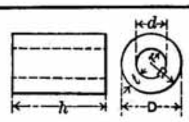
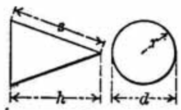
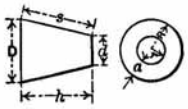
表面積の計算　その2

 **Square**	A = area. $A = s^2$ $A = \frac{1}{2} d^2$ $s = 0.7071\, d = \sqrt{A}$ $d = 1.414\, s = 1.414 \sqrt{A}$
 **Rectangle**	A = area. $A = ab$ $A = a\sqrt{d^2 - a^2} = b\sqrt{d^2 - b^2}$ $d = \sqrt{a^2 + b^2}$ $a = \sqrt{d^2 - b^2} = A \div b$ $b = \sqrt{d^2 - a^2} = A \div a$
 **Parallelogram**	A = area. $A = ab$ $a = A \div b$ $b = A \div a$ Note that dimension a is measured at right angles to line b.
 **Right-angled Triangle**	A = area. $A = \frac{bc}{2}$ $a = \sqrt{b^2 + c^2}$ $b = \sqrt{a^2 - c^2}$ $c = \sqrt{a^2 - b^2}$
 **Acute-angled Triangle**	A = area. $A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}$ If $S = \frac{1}{2}(a + b + c)$, then $A = \sqrt{S(S-a)(S-b)(S-c)}$
 **Obtuse-angled Triangle**	A = area. $A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2b}\right)^2}$ If $S = \frac{1}{2}(a + b + c)$, then $A = \sqrt{S(S-a)(S-b)(S-c)}$
 **Trapezoid**	A = area. $A = \frac{(a + b)\,h}{2}$
 **Trapezium**	A = area. $A = \frac{(H + h)\,a + bh + cH}{2}$ A trapezium can also be divided into two triangles as indicated by the dotted line. The area of each of these triangles is computed, and the results added to find the area of the trapezium.
 **Regular Hexagon**	A = area; R = radius of circumscribed circle; r = radius of inscribed circle. $A = 2.598\, s^2 = 2.598\, R^2 = 3.464\, r^2$ $R = s = 1.155\, r$ $r = 0.866\, s = 0.866\, R$ $s = R = 1.155\, r$
 **Regular Octagon**	A = area; R = radius of circumscribed circle; r = radius of inscribed circle. $A = 4.828\, s^2 = 2.828\, R^2 = 3.314\, r^2$ $R = 1.307\, s = 1.082\, r$ $r = 1.207\, s = 0.924\, R$ $s = 0.765\, R = 0.828\, r$
 **Regular Polygon**	A = area;　n = number of sides. $\alpha = 360° \div n$　$\beta = 180° - \alpha$ $A = \frac{nsr}{2} = \frac{ns}{2}\sqrt{R^2 - \frac{s^2}{4}}$ $R = \sqrt{r^2 + \frac{s^2}{4}}$; $r = \sqrt{R^2 - \frac{s^2}{4}}$; $s = 2\sqrt{R^2 - r^2}$

体積の計算　その1

 Sphere	V = volume; A = area of surface. $V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6} = 4.1888\, r^3 = 0.5236\, d^3$ $A = 4\pi r^2 = \pi d^2 = 12.5664\, r^2 = 3.1416\, d^2$ $r = \sqrt[3]{\frac{3V}{4\pi}} = 0.6204 \sqrt[3]{V}$
 Spherical Sector	V = volume; A = total area of conical and spherical surface. $V = \frac{2\pi r^2 h}{3} = 2.0944\, r^2 h$ $A = 3.1416\, r\,(2h + \frac{1}{2}c)$ $c = 2\sqrt{h\,(2r - h)}$
 Spherical Segment	V = volume; A = area of spherical surface. $V = 3.1416\, h^2\left(r - \frac{h}{3}\right) = 3.1416\, h\left(\frac{c^2}{8} + \frac{h^2}{6}\right)$ $A = 2\pi r h = 6.2832\, rh = 3.1416\left(\frac{c^2}{4} + h^2\right)$ $c = 2\sqrt{h\,(2r - h)}$; $r = \frac{c^2 + 4h^2}{8h}$
 Spherical Zone	V = volume; A = area of spherical surface. $V = 0.5236\, h\left(\frac{3c_1^2}{4} + \frac{3c_2^2}{4} + h^2\right)$ $A = 2\pi r h = 6.2832\, rh$ $r = \sqrt{\frac{c_2^2}{4} + \left(\frac{c_2^2 - c_1^2 - 4h^2}{8h}\right)^2}$
 Spherical Wedge	V = volume; A = area of spherical surface; α = center angle in degrees. $V = \frac{\alpha}{360} \times \frac{4\pi r^3}{3} = 0.0116\, \alpha r^3$ $A = \frac{\alpha}{360} \times 4\pi r^2 = 0.0349\, \alpha r^2$
 Hollow Sphere	V = volume. $V = \frac{4\pi}{3}(R^3 - r^3) = 4.1888\,(R^3 - r^3)$ $= \frac{\pi}{6}(D^3 - d^3) = 0.5236\,(D^3 - d^3)$
 Ellipsoid	V = volume; A = area of surface. $V = \frac{4\pi}{3} abc = 4.1888\, abc$ In an ellipsoid of revolution, or spheroid, where $b = c$: $V = 4.1888\, ab^2$, and $A = \frac{4\pi}{\sqrt{2}} b \sqrt{a^2 + b^2}$
 Paraboloid	V = volume; $V = \frac{1}{2}\pi r^2 h = 0.3927\, d^2 h$ A = area; $A = \frac{2\pi}{3p}\left[\sqrt{\left(\frac{d^2}{4} + p^2\right)^3} - p^3\right]$ in which $p = \frac{d^2}{8h}$
 Paraboloidal Segment	V = volume. $V = \frac{\pi}{2} h\,(R^2 + r^2) = 1.5708\, h\,(R^2 + r^2)$ $= \frac{\pi}{8} h\,(D^2 + d^2) = 0.3927\, h\,(D^2 + d^2)$
 Torus	V = volume; A = area of surface. $V = 2\pi^2 R r^2 = 19.739\, R r^2$ $= \frac{\pi^2}{4} D d^2 = 2.4674\, D d^2$ $A = 4\pi^2 R r = 39.478\, Rr$ $= \pi^2 D d = 9.8696\, Dd$
 Barrel	V = approximate volume. If the sides are bent to the arc of a circle: $V = \frac{1}{12}\pi h\,(2D^2 + d^2) = 0.262\, h\,(2D^2 + d^2)$ If the sides are bent to the arc of a parabola: $V = 0.209\, h\,(2D^2 + Dd + \frac{3}{4} d^2)$
	If d = base diameter and height of a cone, a paraboloid and a cylinder, and the diameter of a sphere, then the volumes of these bodies are to each other as below: Cone: paraboloid: sphere: cylinder = $\frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$

体積の計算　その2

 **Cube**	V = volume. $V = s^3$ $s = \sqrt[3]{V}$
 **Square Prism**	V = volume. $V = abc$ $a = \frac{V}{bc}$ $b = \frac{V}{ac}$ $c = \frac{V}{ab}$
 **Prism**	V = volume; A = area of end surface. $V = h \times A$ The area A of the end surface is found by the formulas for areas of plane figures on the preceding pages. Height h must be measured perpendicular to end surface.
 **Pyramid**	V = volume. $V = \frac{1}{3} h \times$ area of base. If the base is a regular polygon with n sides, and s = length of side, r = radius of inscribed circle, and R = radius of circumscribed circle, then: $V = \frac{nsrh}{6} = \frac{nsh}{6}\sqrt{R^2 - \frac{s^2}{4}}$
 **Frustum of Pyramid**	V = volume. $V = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 \times A_2})$
 **Wedge**	V = volume. $V = \frac{(2a + c)bh}{6}$
 **Cylinder**	V = volume; S = area of cylindrical surface. $V = 3.1416 r^2 h = 0.7854 d^2 h$ $S = 6.2832 rh = 3.1416 dh$ Total area A of cylindrical surface and end surfaces: $A = 6.2832 r(r + h) = 3.1416 d(\frac{1}{2}d + h)$
 **Portion of Cylinder**	V = volume; S = area of cylindrical surface. $V = 1.5708 r^2 (h_1 + h_2) = 0.3927 d^2 (h_1 + h_2)$ $S = 3.1416 r (h_1 + h_2) = 1.5708 d (h_1 + h_2)$
 **Portion of Cylinder**	V = volume; S = area of cylindrical surface. $V = \left(\frac{2}{3}a^3 \pm b \times \text{area } ABC\right)\frac{h}{r \pm b}$ $S = (ad \pm b \times \text{length of arc } ABC)\frac{h}{r \pm b}$ Use + when base area is larger, and − when base area is less than one-half the base circle.
 **Hollow Cylinder**	V = volume. $V = 3.1416 h(R^2 - r^2) = 0.7854 h(D^2 - d^2)$ $= 3.1416 ht(2R - t) = 3.1416 ht(D - t)$ $= 3.1416 ht(2r + t) = 3.1416 ht(d + t)$ $= 3.1416 ht(R + r) = 1.5708 ht(D + d)$
 **Cone**	V = volume; A = area of conical surface. $V = \frac{3.1416 r^2 h}{3} = 1.0472 r^2 h = 0.2618 d^2 h$ $A = 3.1416 r\sqrt{r^2 + h^2} = 3.1416 rs = 1.5708 ds$ $s = \sqrt{r^2 + h^2} = \sqrt{\frac{d^2}{4} + h^2}$
 **Frustum of Cone**	V = volume; A = area of conical surface. $V = 1.0472 h(R^2 + Rr + r^2) = 0.2618 h(D^2 + Dd + d^2)$ $A = 3.1416 s(R + r) = 1.5708 s(D + d)$ $a = R - r$ $s = \sqrt{a^2 + h^2} = \sqrt{(R - r)^2 + h^2}$